Fragmentation and Brittleness: Richard Stacey

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Introduction

- Tough rocks produce larger fragments after blasting. There is a standard model for size estimation.
- Recently a law case arose whereby the standard model didn't correctly determine fragment size.
- Tarasov has defined a novel brittleness index for rocks based on experimental lab tests.

Question: Can this be used to improve estimates for fragment size and distribution?

The Standard Model: Kuz-Ram model

$$x_m = AK^{-0.8}Q^{1/6}(\frac{115}{RWS})^{\frac{19}{20}}$$

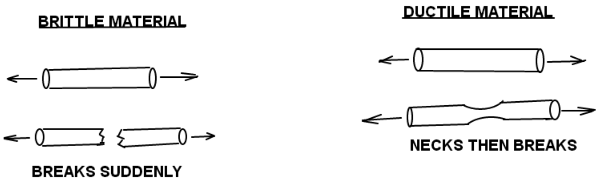
where:
x_m: Mean particle size.
K: Powder factor (kg explosive/m^3)
Q: mass of explosive in hole (kg)
A: a rock 'factor' (0.8-22 !)
RWS: The relative weight strength of the explosive used.
This formula doesn't take into account features of the blast (rock type, bore hole spacing, geometry of the site....)

$$R_x = \exp\left[-0.693(x/x_m)^n\right] \text{with } n = 0.7 - 2$$

Note especially that there is no term in the equation that explicitly takes into account rock properties except A. (eg .brittleness).

Stress Strain Curves: Tarasov (Brittleness Index)

Definition of the Brittle and Ductile rock

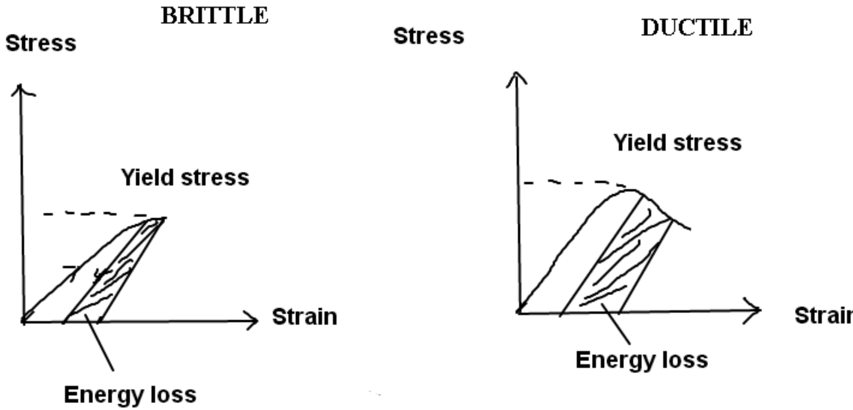


- Brittle rocks crack.
- Ductile rocks 'neck'.
- In general rocks are neither one nor the other.
- Brittleness is used to describe one of the rock characteristics.

Energy loss is greater for a ductile rock.

Stress Strain Curves: Tarasov (Brittleness Index)

The brittleness 'index' (β): Defines the extent to which a rock is 'Brittle/Hard' (Class 2) or 'Ductile' (Class 1) using a stress/strain lab test.



Note the very different shape after the yield stress is exceeded.
The brittleness index β is the area ratio (shaded/total)
Question: Why does this matter?
The Tarasov index quantifies the energy loss associated with stress application!
For a hard/brittle rock much more elastic energy is retained after rupture which means little energy goes into 'Cracking'.

Can this index be used to obtain a better result!

Possible Models

- Energy/Scaling/Statistical model.
- Two mechanistic models.
- Composite models.

A Scaling/Energy Model: Mean particle size

The aim is to improve on the Kuz-Ram model using dimensional analysis.

Model parameters

- Yield stress $Y = \frac{M}{T^2 L}$.
- Brittleness index (β) is dimensionless.
- Energy per unit time per unit volume due to explosive charge $= \epsilon = \frac{M}{T^3 L}$.
- Energy available for fracturing per unit time and volume $= \beta\epsilon$.
- Speed of propagation of elastic wave (primary wave)$= C_P = \frac{L}{T}$
- Mean fragment size$= X_m = L$
- A is a universal constant: applies to all material.

We combine the above parameters to obtain a dimensionally consistent expression for fragment size.

The possible combinations are:

$$X_m = AY^a(\beta\epsilon)^b C_P^c = A(\frac{M}{T^2 L})^a(\beta\frac{M}{LT^3})^b(\frac{L}{T})^c$$

Dimensional analysis: Mean particle size

Dimensionally compatible providing:

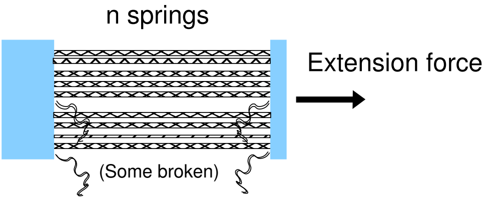
- $L : 1 = -a - b + c$
- $M : 0 = a + b$
- $T : 0 = -2a - 3b - c$

By solving the last equation we will find that $a = 1, b = -1, c = 1$. This gives:

$$***x_m = A\frac{YC_P}{\beta\epsilon}***$$

.
Where $C_P = \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}}$ E is Young's modulus, ν Poisson's Ratio, ρ the density.
Note that this formula includes the important rock properties and is the only combination that makes dimensional sense!

Breaking Springs Model - Simple 1d approach



- Springs (n_0) are stretched by an external force $\mathcal{T}_{ext}$.
- Individual springs have same spring constant (k)
- .... but have different breaking strengths T_s^{crit}.
- External force increased $\Rightarrow$ some springs break
- ..... remaining (n) springs bear the load.

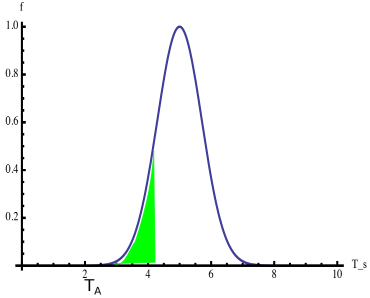
Can model mimic experimental stress-strain results (Class 1 & 2)?

If "Yes", correlate equivalent parameters.

Spring Breakage Distribution

Intact rock corresponds to intact springs, cracks correspond to broken springs. If we assume a normal distribution for breakage:

$$f = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left[\frac{(T_s - \bar{T}_s)^2}{2\sigma^2}\right]}$$



The green shading shows springs that have broken after the application of an individual spring tension T_s.

Equations

Single spring: Tension T_s causes displacement x (initial length l_0):

$$T_s = kx$$

Multiply by number of intact springs $n \Rightarrow$ stress/strain reln:

$$nT_s \equiv \mathcal{T}_{ext} = (nkl_0)\frac{x}{l_0} \equiv E_{eff}\frac{x}{l_0}$$

The effective Young's modulus is defined in terms of k and n.

$$E_{eff} = E_0(\frac{n}{n_0})$$

Springs

Distribution gives number of survivors supporting the load:

$$\frac{n}{n_0} = 1 - \int_0^{T_s} f(T_s) dT_s$$

$$\text{so} \quad E_{eff} = E_0 \left[1 - \int_0^{T_s} f(T_s) dT_s \right]$$

For Normal distribution (the exact result):

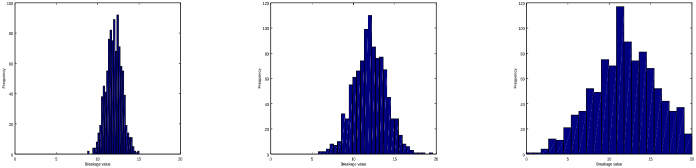
$$E_{eff}(T_s) = E_0 \left[1 - \mathrm{Erf}\left(\frac{T_s - \bar{T}_s}{\sqrt{2\pi}\sigma} \right) \right]$$

and the stress/strain results can be obtained.

$$\mathcal{T}_{ext} = nT_s = E_0 \left[1 - \mathrm{Erf}\left(\frac{T_s - \bar{T}_s}{\sqrt{2\pi}\sigma} \right) \right] \frac{x}{l_0} \qquad (1)$$

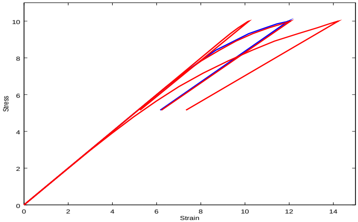
.... unfortunately, don't know n, T_s.

1d model Simulations - 1



Randomly generated normal dsns. of spring breakage tensions for $\sigma = 1, 2, 4$. Left to right - decreasing brittleness. $N = 1000$ springs/bonds.

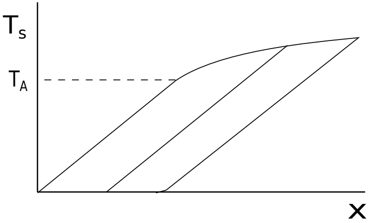
1d model Simulations - 2



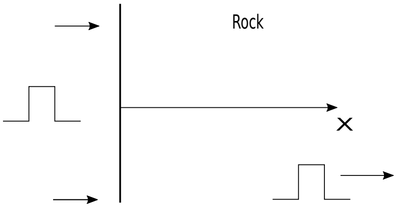
- Corresponding stress-strain relation diagrams. Less brittle $\Rightarrow$ wider curve.
- Agrees well with Class 2 materials
- Distribution of bond breakage/cracking should correlate to particle sizes

Results

- The results asymptote to a T_s with all springs broken
- If applied stress is cycled there is offset, but process repeats
- The rate of approach to the asymptote depends on the distribution width σ.
- One can associate rock characteristics with model parameters (Young's modulus, Yield strength, brittleness)
- The shape is right for Class 2 brittle rocks.
  But Class 1 models aren't covered; k variations needed?

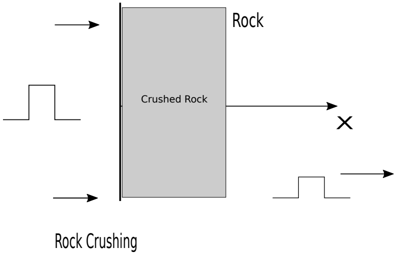


A Continuum State Change Model



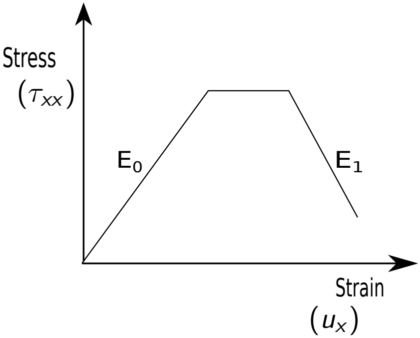
- The end of a semi-infinite rock face (or rod) is impulsively hit.
- If stress levels generated are less than fracture levels T_{crit} then a longitudinal pressure pulse travels away from the face at speed $\sqrt{E_0/\rho}$.
- If stress levels exceed T_{crit} then the rock will partially crush/crack.

Rock Crushing



Note that the transmitted stress wave is reduced due to rock crushing.

Equations



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Newton's Law gives: $\tau_{xx,x} = \rho u_{tt}$, so that with $\tau_{xx} = E^* u_x$, we get

$$E^* u_{xx} = \rho u_{tt}$$

in the (damaged, damaged) and undamaged regions resp.

Ductile and Brittle Rocks

- Note that across the front the equation changes from elliptic to wave type for ductile materials, but not for brittle materials. Interesting!
- Thus in the brittle case the energy decays slowly and the wave travels a great distance. For ductile materials the impulse is quickly damped.
- The extent of damage (cracking) can be assessed using a state change idea. The internal energy of the cracked rock is different.
- The primary aim of the analysis is to determine the speed of travel of the front, the extent of propagation, and the expected fragment size.

Conclusions: The Models

- The energy model. If it works then it could be really important. Needs checking with data
- Springs model: early days but is promising.
- State change model: needs development.